APPENDIX
SOME GROUP THEORISTS

LAGRANGE Joseph (1736-1813) Born in Turin on 25th January 1736. He lived part of his life in Prussia and part in France. He made numerous contributions to astronomy, analysis, number theory and mechanics. He succeeded Euler as the director of the Prussian Academy of Sciences in Berlin, where he stayed for over twenty years producing a large body of work. In 1751 he moved to Paris and became a member of the French Academy. He survived the French Revolution and became the first professor of analysis at the École Polytechnique in 1794. Napoleon awarded him the Legion of Honour. He is known for Lagrange multipliers and his contribution to group theory is Lagrange's Theorem. Group theory was yet to be born, but he stated that if a polynomial in n variables has its variables permuted in all possible ways the number of distinct polynomials so obtained is a factor of $n!$ In effect this is equivalent to the fact that Lagrange's Theorem holds for the symmetric groups.

ABEL Niels Hendik (1802-1829) Born in Norway on 5th August 1802. He entered the Royal Frederick University in 1821. There he started his work on the solvability of the quintic polynomial by radicals. At first he thought he had a solution, expressing the zeros in terms of the coefficients using square roots, cube roots, fourth roots and fifth roots. The two professors in Oslo were convinced that his solution was correct. A Danish mathematician who examined his manuscript could find no error, but was sceptical that an unknown could solve such a long-standing problem. He asked Abel to give a numerical example and, while trying to provide such an example, Abel discovered an error in his paper. He then went on, at the age of 19, to prove that the general quintic was insoluble by radicals. His picture appears on a Norwegian stamp as well as on a 500 kroner banknote.

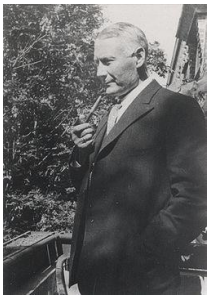
GALOIS Évariste (1811-1832) Born in Bourg-la-Reine in France on 25th October 1811. In 1828, having failed the entrance exam for the École Polytechnique, he enrolled in a school that was much inferior. He later attended the École Normale where his examiner reported that "this pupil is sometimes obscure in expressing his ideas, but he is intelligent and shows a remarkable spirit of

research". He studied the theory of polynomial equations and developed his far-reaching theory that was able to tell which equations could be solved by radicals. This was done in terms of groups of substitutions on the roots, what we would now call permutation groups. He submitted his memoir to the French Academy on several occasions but it was either rejected or mislaid. On the last occasion it was submitted to Fourier to be considered for the Grand Prix, but Fourier died soon after and the memoir was lost. He became a radical republican during the reign of Louis Phillipe and his political views led him to be expelled from the École Normale and to spend time in prison, where he continued his research. He was challenged to a duel on May 30th 1832. The reasons are not clear, but a letter he wrote a few days before his death suggests it might have involved a broken love affair. The night before the duel he sat up all night writing out an outline of his discoveries and hoped that one day someone would be able to "decipher all this mess". He was shot in the abdomen and died the next day. His memoir was discovered many years later and in 1846 his discoveries were finally published. Group theory was born, though it was some time later before it was detached from the theory of equations and was expressed in terms of abstract permutations. The final stage of abstraction, when the group axioms were distilled, did not occur until 1882.

FROBENIUS Ferdinand (1849-1917) Born in Berlin on 26 October 1849. He studied differential equations under Weierstrass and taught in Berlin for a few years before being given an appointment in Zurich. In 1893 he returned to Berlin where he was elected to the Prussian Academy of Sciences. In the second half of his career he concentrated on group theory and he gave the first proof of the first Sylow theorem for abstract groups (early proofs relied on the group being a permutation group). He proved that if n divides the order of G then the number of solutions to $x^n = 1$ is a multiple of n and asked whether, if it is exactly n, the set of such solutions is a subgroup. This remained unsolved until 1991 when it was shown to be true for all finite groups. He is more famous for having created the theory of group characters.

BURNSIDE William (1852-1927) Born in London on 2nd July 1852. He lectured at Cambridge from 1875-1885 where he was a famous rower. He was then appointed as professor of Mathematics at the Royal Naval College in Greenwich. His early work was in applied mathematics and earned his election as a fellow of the Royal Society. He then turned his attention to finite groups, and especially their

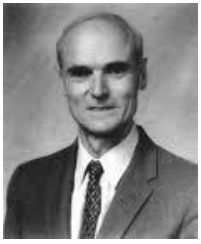
representations. He is best known for his $p^a q^b$ theorem that states that every finite group, whose order is divisible by only 2 primes, is soluble. This was proved using group characters, and it was only recently that it was able to be proved without character theory. His book *Theory of Groups of Finite Order*, published in 1897, became the standard text on group theory for many decades. Burnside is also known for the Burnside problem. Is there a finitely generated group, where every element has finite order, that is in fact infinite. There is. The restricted Burnside problem, for any m, n is "is there an infinite group on m generators that satisfies $g^n = 1$ for all $g \in G$?" For $n \leq 4$ the answer is "no". However the answer is not known for $n = 5$, but if n is odd and greater than 665 the answer is "yes".



NIELSEN Jacob (1890-1959) Born on an island that forms part of Denmark on 15th October 1890. After completing his doctorate from the University of Kiel he was drafted into the German Imperial Navy where he was assigned to coastal defence. In 1920 he took a position at the Technical University of Breslau. The next year he published a proof that every subgroup of a finitely generated free group is free. Otto Schreier extended this to free groups of infinite rank and the theorem is now known as the Nielsen-Schreier theorem.

He then moved to Copenhagen. He served with UNESCO from 1952 to 1958.

HIRSCH Kurt (1906-1986) Kurt was born in Germany on 12th January 1906. He obtained his first PhD in the logical foundations of mathematics. He went to England in 1934 and obtained a second doctorate at Cambridge under Philip Hall, the subject being polycyclic groups. During the war he was interred as "an enemy alien" on the Isle of Man, where he worked as a cook. When he was released he moved to the University of Newcastle. In 1951 he moved to Queen Mary College in the University of London where he built up a strong centre for algebra. In addition to polycyclic groups he worked on locally nilpotent groups and automorphisms of groups.



COXETER Harold (1907-2003) He was born in London on 9th February 1907 but spent most of his life in Canada. He worked for 60 years at the University of Toronto, mainly in geometry which he approached using algebra. He is considered one of the greatest geometers of the 20th century. He knew Maurits Escher and helped to inspire much of Escher's artistic work. In 1928, with the

collaboration of Todd, he developed an algorithm for turning a presentation of a group into a group table.

NEUMANN Bernhard (1909-2002)

Born in Germany on 15th October 1909. He studied at the University of Berlin where he was a fellow student of Kurt Hirsch. He obtained his PhD and went to England in 1933. There he obtained a second PhD. His examiners were Philip Hall and M.H.A Newman. A normal requirement, after having a PhD accepted, is an oral exam, which is not always taken too seriously. Bernhard was asked two questions "would you prefer beer or wine for lunch?" and "do you take your coffee black or white?". After being awarded this second PhD he stayed in Cambridge for two years. Just before leaving Germany he met another very bright student in the cafeteria. She was Hanna von Caemmerer. They went for bicycle rides and on one occasion her brakes failed and she fell off and grazed her leg. Hanna recalled that Bernhard first straightened her bent wheel before he went over to ask how she was! She joined him in England just before war broke out and they got married. They had jobs in different cities but eventually they managed to both get jobs in Manchester. In the 1950s the Australian National University was very young and was expanding. A chair was offered to Bernhard. He wrote back saying that he and Hanna were like a butler/cook husband-wife team.

"You have to hire us both." She had quite a strong reputation in her own right, so this was arranged. In Canberra Bernhard set up a department that was, at the time, one of the leading group theory departments in the world. Hanna died in 1971 and Bernhard continued inspiring generations of group theorists until he retired. Then he worked for the CSIRO for three years and undertook many other projects. He was a familiar sight, up until his death, cycling around Canberra. In his latter years he moved very slowly, wobbling left and right, and became somewhat of a traffic hazard, but he cycled up till his death at the age of 92. Bernard and Hanna both worked in the area of varieties of groups.

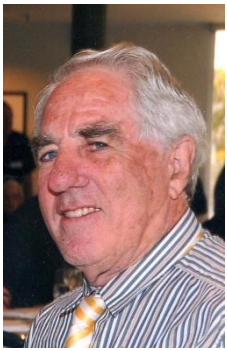
KAPLANSKY Irving (1917-2006) Irving was born in Toronto on 22nd March 1917, shortly after his parents migrated from Poland. Irving developed musical skills at an early age and his parents assumed that he would make music his career. He played with several bands, in nightclubs and other venues. This helped him to finance his studies. In 1941 he obtained his PhD from Harvard and worked there until 1944. In 1945 he moved to the University of Chicago where he remained until he retired in 1984. He then moved to Berkeley in California to become the director of the Mathematical Sciences Research Institute. His

research interests included ring theory, group theory, Galois theory and number theory. Among his many publications is a tiny, but influential, book on infinite abelian groups. He has written many songs about mathematics. One was *A Song About Pi* where he turned the first 14 decimal places into musical notes. Another of his songs, to the tune of *That's Entertainment*, is:

The fun when two parallels meet
Or a group with an action discrete
Or the thrill when some decimals repeat,
That's mathematics.

A nova, incredibly bright,
Or the speed of a photon of light,
Andrew Wiles, proving Fermat was right,
That's mathematics.

COOPER, Christopher [1942 -]



The only way I could ever get into a list of group-theorists is by compiling the list myself. But I always find it interesting to find out about the author of any book I read, so here goes.

I studied at the University of Sydney and obtained my PhD at the University of London (Queen Mary College). Kurt Hirsch was head of department, and I did a course on infinite abelian groups

under Irving Kaplansky who was there for a semester. I published a few papers in group theory but became more interested in the teaching of mathematics, and in particular deciding what should be taught. Taking up a position at Macquarie University in its early days (I taught the first third year algebra course in 1969) I was able to lay the foundations for courses in that area.

I have a strong interest in the interaction between mathematics and religion as a result of my being a lay preacher in the Uniting Church. Other interests include literature – especially the Brontë sisters – bell ringing, and narrow-boating. I married a Dutch girl and we had four children. Sadly Elisabeth has now died and so I am a retired widower.

PRAEGER, Cheryl [1948 -]

Cheryl Praeger is one of the top mathematicians in Australia – not just in the category of women mathematicians. She attended high school at the Brisbane Girls Grammar School.

After graduating she consulted a vocational guidance consultant, explaining that she wanted to study maths. He tried to dissuade her because he said that in his experience girls don't do well in mathematics. Perhaps she could become a nurse. When she persisted he said that she could become an engineer. There was maths in that, but not enough for her.

She studied mathematics and physics at the University of Queensland and then took a Commonwealth scholarship to study at the University of Oxford, in St Anne's College. She decided at that point that she wanted to study algebra. She obtained her doctorate in 1973 and became a research fellow at the Australian National University in Canberra.

She has a special interest in encouraging women, not just to study maths but to pursue a career in mathematics. She married a statistician, John Henstridge in 1975 and has two sons. She was offered a position at the University of Western Australia where she became a professor, and head of department. She still works there. She has received many honours over the years, including a Doctor of Science degree (much higher than a PhD). In 2021 she was awarded the honour of Companion of the Order of Australia. She is a fellow of the Australian Academy of Sciences.

Her research interests have been mainly in the area of group theory, especially the theory of permutation groups, and combinatorics. Her other interests include sailing, hiking and cycling. She is an Associate in Music (piano) and plays at the Nedland Uniting Church where she is a member, and has been an elder.

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LIST OF TITLES

GENERAL
- The Mathematics At The Edge Of The Rational Universe

ELEMENTARY
- Basic Mathematics
- Concepts of Algebra
- Concepts of Calculus
- Elementary Algebra
- Elementary Calculus

1st YEAR UNI
- Techniques of Algebra
- Techniques of Calculus
- Matrices

2nd YEAR UNI
- Linear Algebra
- Languages & Machines
- Discrete Mathematics

3rd YEAR UNI
- Group Theory volume 1
- Group Theory volume 2
- Galois Theory
- Graph Theory
- Number Theory
- Geometry
- Topology
- Set Theory

POSTGRADUATE
- Ring Theory
- Representation Theory
- Quadratic Forms
- Group Tables vol 1
- Group Tables vol 2